

# pQCD at High Energy

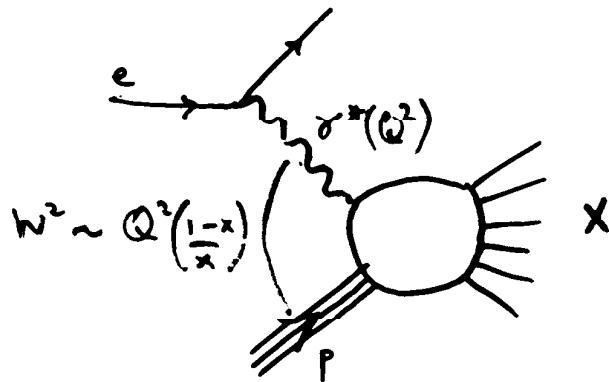
DIS 7+

Chicago

Apr 1997

Richard Ball.

(in inelastic ep scattering)



At high  $Q^2$ ,

moderate  $x$ :

$\left\{ \begin{array}{l} \text{mass factorization (ope)} \\ + \text{ren grp invariance } (\Rightarrow \alpha_s(Q^2)) \\ + \text{asymptotic freedom} \end{array} \right.$

$\Rightarrow$  self consistent pQCD evolution in  $Q^2$

Gross, Wilczek,  
Georgi, Politzer 1973

At high  $W^2$  (small  $x$ ),

moderate  $Q^2$ : either: Regge theory,  $P, x^{-0.08}$  etc  
1960s

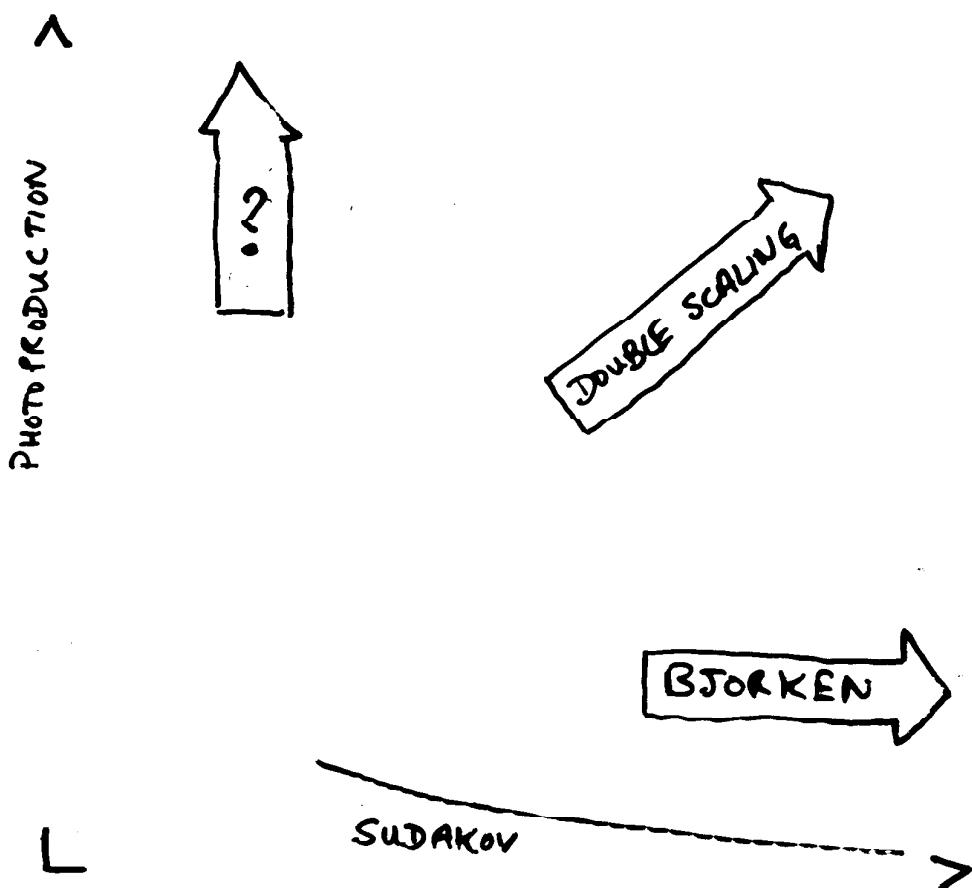
or:

\* data \*

DIS 96  
(Rome)

$\left\{ \begin{array}{l} \text{pQCD: summation of } \ln/x, P/P, x^{-4\ln 2\bar{\alpha}_s} \text{ etc.} \\ \text{or adding sums of } \ln/x \text{ to evolution in } Q^2 \\ \text{BFKL, 1977} \\ \text{Catani et al 1993.} \end{array} \right.$

Imagine a world without a IP ....



$Q^2/\Lambda^2 \rightarrow \infty, S^2/\Lambda^2$  fixed

$\left\{ \begin{array}{l} \text{Mass factorization} \\ + \text{ren. grp. resummation of } \ln(Q^2/\Lambda^2) \\ + \text{asymptotic freedom } (\Rightarrow \alpha_s(Q^2)) \\ \Rightarrow \text{self-consistent pQCD evolution in } Q^2 \end{array} \right.$

$Q^2/\Lambda^2 \rightarrow \infty, S^2/\Lambda^2 \rightarrow \infty$

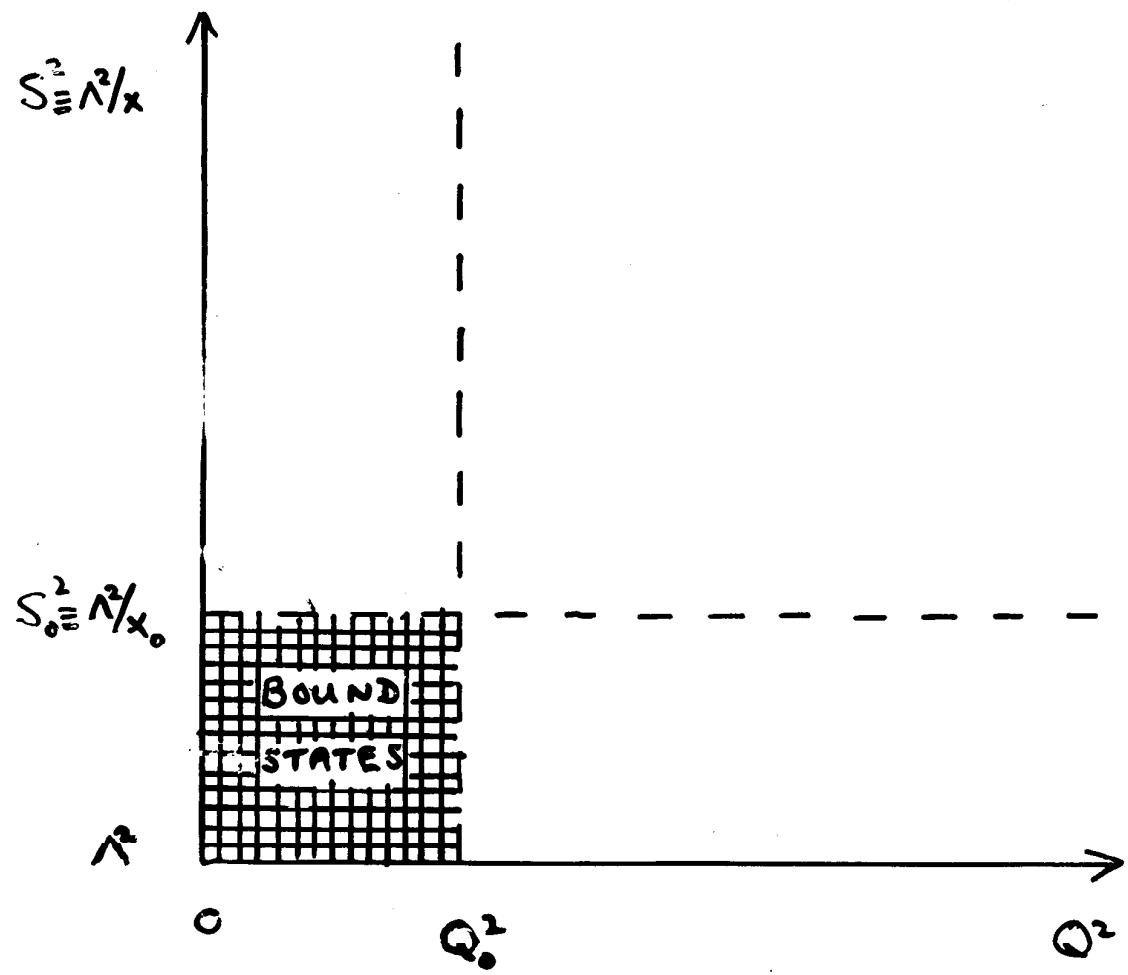
pQCD evolution in  $Q^2$  at small  $x$   
 $\Rightarrow$  DAS

$Q^2/\Lambda^2$  fixed,  $S^2/\Lambda^2 \rightarrow \infty$

Factorization Thm  
 Ren. grp. resummation of  $\ln(1/x)$

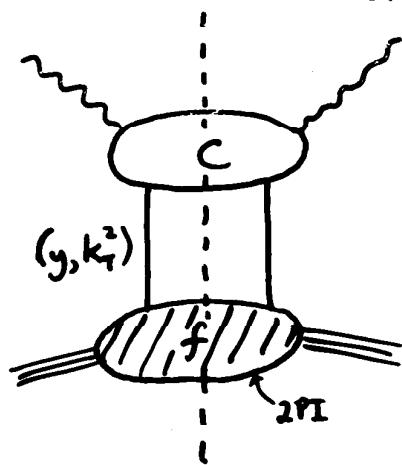
?

The  $x - Q^2$  plane



## Double Factorization

or 'k<sub>q</sub>-factorization', Cetani et al.



2PR correction to xsec

$$\sigma(x, Q^2) = \int_x^1 \frac{dy}{y} \int_0^{w^2/4} \frac{dk^2}{k^2} C\left(\frac{y}{x}, \frac{Q^2}{k^2}; \alpha_s(\mu)\right) f(y, k^2; \mu)$$

kinematics      renormalization  
 scale.

integrate over      coefficient fn.      'unintegrated'  
 momenta of internal      lines      pdf

Proof: by a projection onto color + spin singlet (Cetani - Hartmann)  
 fixed scale (in axial gauges)

Write

$$S^2 \equiv \Lambda^2/x, \quad \ell^2 \equiv \Lambda^2/y$$

Extend  
Integration  
limits (O.K.)

$$\sigma\left(\frac{S^2}{\mu^2}, \frac{Q^2}{\mu^2}; \mu\right) = \underbrace{\int_0^\infty \frac{d\ell^2}{\ell^2}}_{\text{fictitious } \mu-\text{dep}} \int_0^\infty \frac{dk^2}{k^2} C\left(\frac{S^2}{\ell^2}, \frac{Q^2}{k^2}; \alpha_s(\mu)\right) f\left(\frac{\ell^2}{\mu^2}, \frac{k^2}{\mu^2}; \mu\right)$$

long. trans.  
 mom. mom.

Undo double convolution by Mellin transforms:

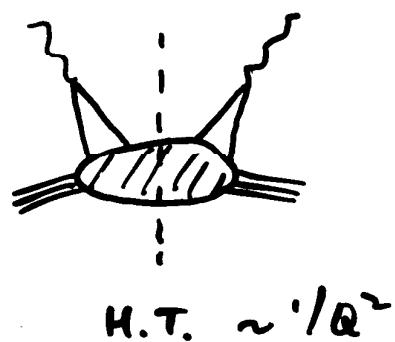
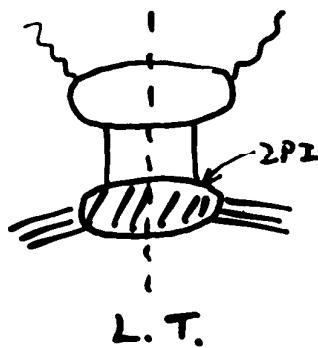
$$\sigma_{NM}(\mu) \equiv \int_0^\infty \frac{dS^2}{S^2} \left(\frac{\Lambda^2}{S^2}\right)^N \int_0^\infty \frac{d\ell^2}{\ell^2} \left(\frac{\Lambda^2}{\ell^2}\right)^M \sigma\left(\frac{S^2}{\mu^2}, \frac{Q^2}{\mu^2}; \mu\right) \text{ etc}$$

then

$$\sigma_{NM}(\mu) = C_{NM}(\alpha_s(\mu)) f_{NM}(\mu)$$

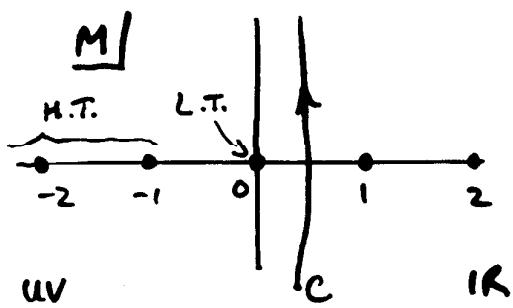
# Mass Factorization ( $Q^2 \rightarrow \infty$ )

At large  $Q^2$ , 2PI graphs down by  $\sim 1/Q^2$  (Weinberg's Thm)



So

$$\sigma_N(Q^2/\mu^2; \mu^2) \sim \int_C \frac{dM}{2\pi i} \left(\frac{Q^2}{M^2}\right)^m C_{NM}(\alpha_s(\mu^2)) f_{NM}(\mu^2) + O(1/\alpha^2)$$



Leading contribution as  $Q^2 \rightarrow \infty$  from  $M \sim C$

$$C_{NM}(\omega) = \sum_{m=-\infty}^{\infty} C_N^m(\omega) M^{-m-1}$$

$\uparrow$   
 $O(\omega^m)$

$(\ln Q^2/k^2)$

$$f_{NM}(\mu^2) = \sum_{m=0}^{\infty} f_N^m(\mu^2) M^m$$

$\downarrow$   
No logs :: 2PI :  $f_N(k) \sim \frac{1}{k^2}$

$$f_N^m(\mu^2) = \frac{1}{m!} \int_0^\infty \frac{dk^2}{k^2} (\ln p^2/k^2)^m f_N(k^2; \mu^2)$$

Leading logs: extra power of  $M$  in  $f_{NM} \Rightarrow$  extra power of  $\alpha_s$  in  $C_{NM}$

so

$$\boxed{\sigma_N(Q^2/\mu^2; \mu^2) = C_N(Q^2/\mu^2; \alpha_s(\mu^2)) F_N(\mu^2) + O(1/\alpha^2)}$$

Extend to all orders in  $\alpha_s$  by choosing suitable factorization schemes:

then  $F_N(\mu^2) = f_N^0(\mu^2) + C_N^0(\alpha_s(\mu^2))^{-1} \sum_{m=1}^{\infty} C_N^m(\alpha_s(\mu^2)) f_N^m(\mu^2)$

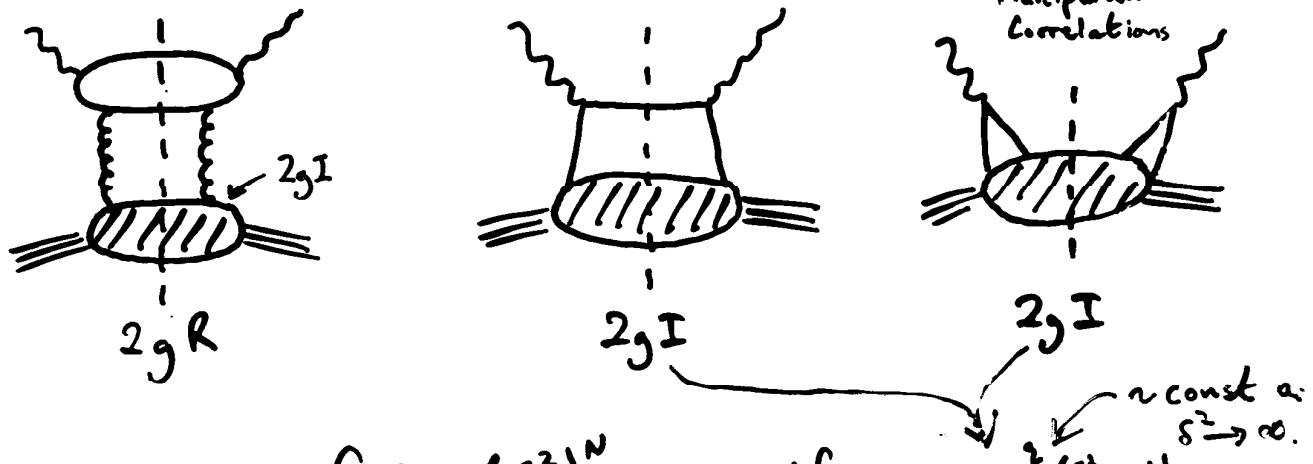
Integrated PDF

## Energy Factorization

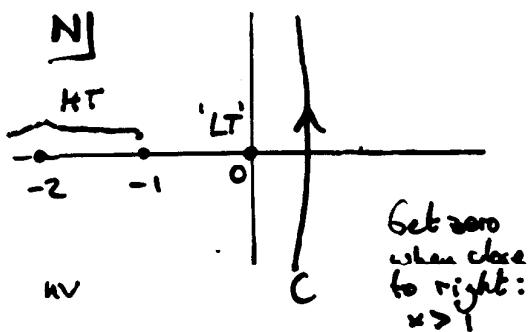
$(S^2 \rightarrow \infty)$

$S^2 \leftrightarrow Q^2$   
 $N \leftrightarrow M$

At large  $S^2$  (ie small  $x$ ) only 2 gluon reducible graphs have log sing: others behave at most as a constant:



$$\text{So } \sigma_n(S^2_{\mu^2}; \mu^2) = \int_C \frac{dN}{2\pi i} \left(\frac{S^2}{\lambda^2}\right)^N C_{NM}(\alpha_s(\mu^2)) f_{nm}(\mu^2) + \sigma_n^2 \left(\frac{S^2}{\mu^2}; \mu^2\right)$$



Leading contribution as  $S^2 \rightarrow \infty$  from  $N \sim 0$

$$C_{NM}(\alpha) = N \sum_{n=0}^{\infty} C_n^{(0)} N^{-n-1}$$

$\xrightarrow{\text{quark box}} \quad \xrightarrow{-\infty} \quad \xrightarrow{O(\alpha^n)}$

$$(1 \ln S^2 / e^2)^n$$

$$f_{nm}(\mu^2) = N^{-1} \sum_{n=0}^{\infty} f_n^{(0)}(\mu^2) N^n$$

No logs  $\therefore 2gI : f_n(l^2) \underset{l^2 \gg n}{\sim} \text{const}$

$$f_n^{(0)} = \frac{1}{n!} \int_0^\infty \frac{dt^2}{t^2} (\ln \mu^2 / t^2)^n \frac{\partial}{\partial t^2} f_n(t^2 / \mu^2; \mu^2)$$

Use same leading log argument as for mass fact: gives

Energy Factorization	$\sigma_n(S^2_{\mu^2}; \mu^2) = C_n(S^2_{\mu^2}; \alpha_s(\mu^2)) F_n(\mu^2) + \sigma_n^2 + O(1/S^2)$
----------------------	-------------------------------------------------------------------------------------------------------

True to all orders  $\sim \alpha_s$  by choosing suitable energy factorization scheme

$$F_n(\mu^2) = f_n^0(\mu^2) + C_n^0(\alpha_s(\mu^2)) \sum_{n=1}^{\infty} C_n(\alpha_s(\mu^2)) f_n^{(0)}(\mu^2)$$

Integrated pdf

large  $S^2$

Energy factorization

$$\sigma_m(S_{\mu^2}^2; \mu^2) = C_m(S_{\mu^2}^2; \alpha_s(\mu^2)) F_m(\mu^2) + \dots$$

$$\left( \mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} + \gamma_m(\alpha_s) \right) C_m(S_{\mu^2}^2; \alpha_s(\mu^2)) = 0$$

$$\mu^2 \frac{d}{d\mu^2} F_m(\mu^2) = \overleftarrow{\gamma_m(\alpha_s(\mu^2))} F_m(\mu^2)$$

'Transverse evolution'

'Longitudinal evolution'

$$\Rightarrow \sigma_m(S_{\mu^2}^2; \mu^2) = \underbrace{C_m(1; \alpha_s(S^2))}_{\text{coupling runs with } S^2} \underbrace{\Gamma_m(S^2, \mu^2)}_{F_m(S^2)} F_m(\mu^2) + \sigma_m^2 + O(1/S)$$

$$\Gamma_m(S^2, \mu^2) = \exp \int_{\alpha_s(\mu^2)}^{\alpha_s(S^2)} \frac{dx}{\beta(x)} \gamma_m(x)$$

At high energies QCD is asymptotically free:  
the coupling runs with  $\delta^2 = \Lambda^2/x$

Even if  $Q^2$  is small!

$\rightarrow$   
 $\rightarrow$

# Renormalization Group Invariance

Wilson  
G-W-G-P

Double

factorization

$$\xrightarrow{\text{large } Q^2} \text{Mass factorization} \quad \tilde{\sigma}_N(Q^2/\mu^2; \mu^2) = C_N(Q^2/\mu^2; \alpha_s(\mu^2)) F_N(\mu^2) + \dots$$

Ren. grp. inv  
of  $\times$  sec

$$\mu^2 \frac{d}{d\mu^2} \sigma = 0$$

$$\xrightarrow{\text{ren. grp. eqn.}} \left( \mu^2 \frac{d}{d\mu^2} + \beta(\alpha_s) \frac{d}{d\alpha_s} + \gamma_N(\alpha_s) \right) C_N(Q^2/\mu^2; \mu^2) = 0$$

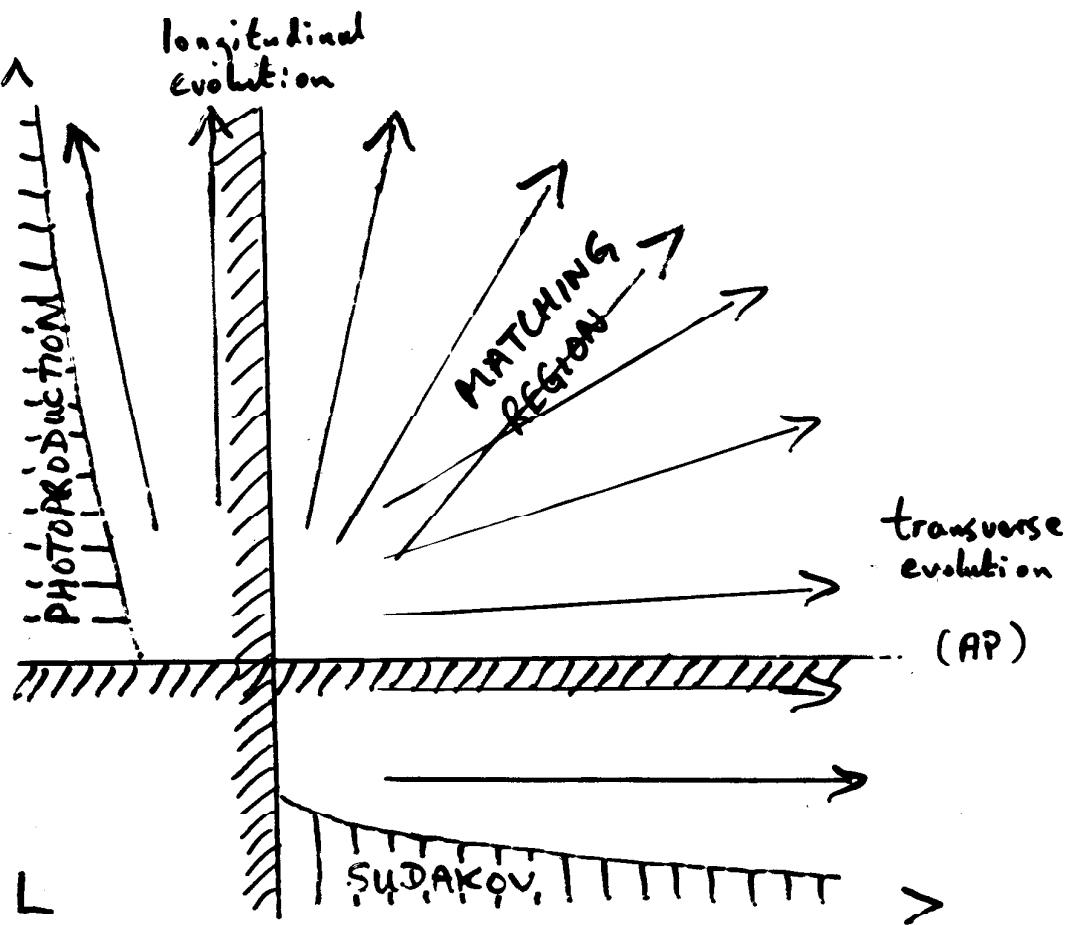
$$\xrightarrow{\text{Eval. eqn.}} \mu^2 \frac{d}{d\mu^2} F_N(\mu^2) = \gamma_N(\alpha_s(\mu^2)) F_N(\mu^2)$$

$\gamma$  anomalous dim

Solving the  
ren grp eqn.

$$\tilde{\sigma}_N(Q^2/\mu^2; \mu^2) = \overbrace{C_N(1; \alpha_s(Q^2))}^{\substack{\text{coupling runs} \\ \text{with } Q^2}} \overbrace{\Gamma_N(Q^2, \mu^2)}^F F_N(\mu^2) + O(1/Q^2)$$

$$\text{where } \Gamma_N(Q^2, \mu^2) = \exp \int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)} \frac{d\alpha}{\beta(\alpha)} \gamma_N(\alpha)$$



Transverse evolution

$$\frac{Q^2 \partial}{\partial Q^2} F_N(Q^2) = \gamma_N(\alpha_s(Q^2)) F_N(Q^2)$$

Evolve from (non-part) b.c.

$$F_N(Q_0^2)$$

Longitudinal evolution

$$\frac{s^2 \partial}{\partial s^2} F_m(s^2) = \gamma_m(\alpha_s(s^2)) F_m(s^2)$$

Evolve from (non-part) b.c.

$$F_m(S_0^2)$$

LO

 $\overbrace{\text{MN-duality}}$ 

Assume for  
simplicity coupling  
is fixed.

Longitudinal evolution

$$\frac{\partial}{\partial \ln S^2} \sigma_m(s) = \bar{\alpha}_s \chi(m) \sigma_m(s)$$

$\uparrow_{C_{\text{res}}/\pi}$

Transverse evolution (gluons only)

$$\frac{\partial}{\partial \ln Q^2} \sigma_n(Q) = \bar{\alpha}_s \phi(n) \sigma_n(Q)$$

$\uparrow_{\delta \sigma_n}$

Mellin tfm:

$$N \sigma_{nm} = \bar{\alpha}_s \chi(n) \sigma_{nm}$$

Valid for  $N \ll 1$ 

$$M \sigma_{nm} = \bar{\alpha}_s \phi(n) \sigma_{nm}$$

Valid for  $M \ll 1$ 

$\nwarrow$  When  $N \ll 1$  and  $M \ll 1$   
both eqns give  
 $N M = \bar{\alpha}_s$

Each eqn. defines a trajectory in  $m-N$  plane, which reaches close to origin.

Duality: define "leading sing. anom. dim."

$$l = \frac{\bar{\alpha}_s}{N} \chi\left(\gamma_T^{ls}\left(\frac{\bar{\alpha}_s}{N}\right)\right)$$

 $\uparrow$  'Lipshiz anom. dim.'

$$l = \frac{\bar{\alpha}_s}{M} \phi\left(\gamma_L^{ls}\left(\frac{\bar{\alpha}_s}{M}\right)\right)$$

Then evolution eqns become

$$M \sigma_{nm} = \gamma_T^{ls}\left(\frac{\bar{\alpha}_s}{N}\right) \sigma_{nm}$$

$$N \sigma_{nm} = \gamma_L^{ls}\left(\frac{\bar{\alpha}_s}{M}\right) \sigma_{nm}$$

i.e. transverse evoln:

anom. dim. sum. leading sing

i.e. longitudinal evoln:

anom. dim. sum. leading sing

Gives powerful cross checks on transverse and longitudinal anom. dims.

N.B. This duality is exact: no 'higher twist projection'.

## Calculation of LO Hadronic Dimension

Write longitudinal evolution eqn in 'Altarelli-Parisi' form:

$$S^2 \frac{\partial}{\partial S^2} F(Q^2, S^2) = \underline{\omega_S(S^2)} \int_0^\infty \frac{dk^2}{k^2} \underbrace{P(Q^2/k^2; \alpha_s(s^2))}_{\text{Splitting function.}} F(k^2, S^2)$$

where  $\underline{\omega_n(\alpha)} = \underline{\alpha} \int_0^\infty \frac{dk}{k} \underline{\kappa^n} P(k; \alpha) \quad \text{etc.}$

Calculate splitting function using N-N approx of Altarelli-Parisi in the infinite momentum frame:

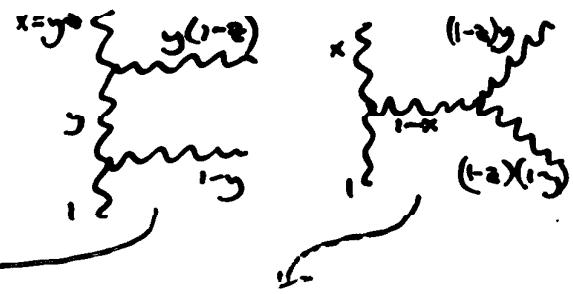
For transverse evol.  
pick out coeff of  
 $dk^2/k^2$

Emission from  
t-channel  
partons only

$k^2 \left\{ \begin{array}{l} \text{sum} \\ \text{for } \{ \end{array} \right\}$

For longitudinal evol.  
pick out coeff of  
 $dy/y$

Emission from  
both t-channel  
and s-channel:  
gluons only



- Amplitude  $\sim 2g^2 \left\{ \frac{(k'-k)^2}{|k-k'|^2} \frac{(-k')}{z} + \frac{y}{|k'|^2} \frac{k''}{z} \frac{k''}{z} \right\}$   
for  $z \ll 1$ , i.e.  $x \gg y$        $k = k_x + ik_\perp$

- Square,  $\rightarrow$  sum over helicities  $\rightarrow$  colors:

$$d\sigma^{(0)} = \frac{dy}{y} \left\{ \frac{dk}{k} \underbrace{\frac{2C_A}{2\pi} \frac{1}{|k-x|}}_{\text{Real part of } P(k)} \right\} d\sigma^{(0)} \quad \left| \frac{x-y}{z} \right|^2$$

$$\kappa = |k'^2/|k|^2|$$

- Virtual contributions  $\propto \delta(1-\kappa)$ : regularise singularity at  $\kappa=1$ .

$$P(k) = \left[ 2 \frac{1}{1-k} \Big|_+ \Theta(1-k) - P \frac{1}{1-k} + c \delta(1-k) \right]$$

$$\underline{\omega_n(\alpha)} = \underline{C_A \alpha_S} X(n)$$

$$X(n) = 2\psi(1) - \psi(n) - \psi(1-n)$$

## High Energy Factors

- Inclusion of Quarks:

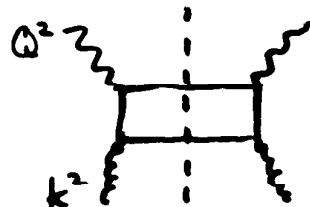
$$\begin{array}{l} \text{gluon} \rightarrow \text{gluon} \\ \text{quark} \rightarrow \text{gluon} \\ \text{gluon} \rightarrow \text{quark} \\ \text{quark} \rightarrow \text{quark} \end{array} \quad \left. \begin{array}{l} \text{generates } \ln^{1/2} \\ \text{no } \ln^{1/2} \end{array} \right\}$$

So at leading logs: initial quark immediately turns into a gluon, so absorb into gluon distn.

There is only one evolving parton (the gluon)

Can choose factorization schemes in which this is true to all orders.

- Coefficient functions (LO)



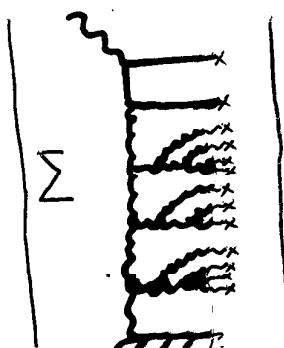
Take Mellin fms,  
let  $N \rightarrow 0$  (leading  $\ln^{1/2}$ )

Find e.g.  $C_M^2(1, \alpha_s) = \frac{\alpha_s}{2\pi} \text{Tr} \left[ \frac{8}{1-N} \left( \frac{3}{2} + \frac{1}{N(N-1)} \right) \right]$  cf Catani et al.  
(for massless quarks)  $C_M^L(1, \alpha_s) = \frac{\alpha_s}{2\pi} \text{Tr} \left[ \frac{8}{1-N} \right]$  universal factor:  
absorb into gluon distn.

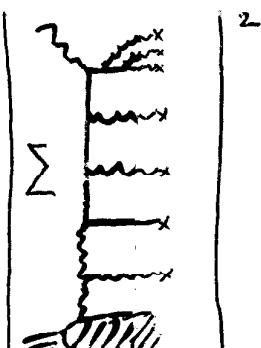
Only ratios of coeff fns have physical significance.

- Can check explicitly that in W-W approx gluon emissions iterate, to generate all cut rainbow-ladder graphs (+ virtual loops) (crossed)

Leading logs at fixed coupling.



4 transverse case



Write longitudinal evolution eqn in 'Altarelli-Parisi' form:

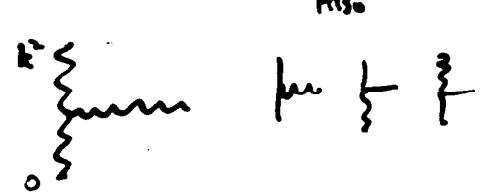
$$S^2 \frac{\partial}{\partial S^2} F(Q^2, S) = \frac{\alpha_s(S)}{2\pi} \int_0^\infty \frac{dk^2}{k^2} P(Q^2/k^2; \alpha_s(S)) F(k^2, S)$$

where  $\gamma_n(\omega) = \frac{1}{2\pi} \int_0^\infty \frac{dk}{k} k^n P(k; \omega) \text{ etc.}$  splitting function.

Calculate splitting function using N-W approx cf Altarelli-Parisi  
in the infinite momentum frame:

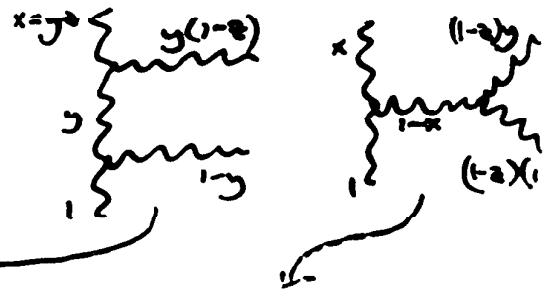
for transverse evol.  
pick out coeff of  
 $dk^2/k^2$

Emission from  
t-channel  
partons only



for longitudinal evol.  
pick out coeff of  
 $dy/y$

Emission from  
both t-channel  
and s-channel:  
gluons only



- Amplitude  $\sim 2g^2 \left\{ \frac{(k'-k)}{|k-k'|^2} \frac{(-k)}{z} + \frac{y}{|k'|^2} \frac{k'}{z} \frac{k''}{z} \right\}$  for  $z \ll 1$ , i.e.  $x \ll y$   $k = k_x +$

- Square,  $\rightarrow$  sum over helicities  $\rightarrow$  colors:  $d\sigma^{(0)} = \frac{dy}{y} \left\{ \frac{dk}{k} \underbrace{2C_A \frac{1}{2\pi |1-k|}}_{\text{Real part of } P(k)} \right\} d\sigma^{(0)}$   $k = |k'^2/|k|^2|$

- Virtual contributions  $\propto \delta(1-k)$ : regularise singularity at  $k=1$ .

$$P(k) = \left[ 2 \frac{1}{1-k} \Big|_+ \Theta(1-k) - P \frac{1}{1-k} + C \overline{\delta(1-k)} \right]$$

$$\gamma_n(\omega) = C_n \underline{\alpha_s} \chi(n)$$

$$\chi(n) = 2\psi(1) - \psi(n) - \psi(1-n)$$

## Solution of (Longitudinal) Evolution Eqn

$$\frac{\partial}{\partial \ln \frac{Q^2}{x}} G_m = \frac{4C_n}{\beta_0} \frac{1}{\ln^2 x} \chi(n) G_m$$

$\gamma = \sqrt{\frac{\beta_0}{\alpha_s}}$

$$\therefore F_2(x, Q^2) = \int_C \underbrace{\frac{dM}{2\pi i}}_{\text{inverse Mellin}} \left( \frac{Q^2}{\lambda^2} \right)^M \underbrace{\frac{1}{2} \langle e^2 \rangle C_n^2 (1; \alpha_s(s))}_{\text{coeff. fn.}} \underbrace{\left( \frac{\ln^2 x}{\ln^2 x_0} \right)}_{\text{evol. factor}} \underbrace{G_m^0 + F_2^0(Q^2)}_{\text{bdy. condition}}$$

Need an input dist'n: take e.g.

$$G^0(Q^2) = G_0 \frac{Q^2}{Q^2 + \lambda_0^2} : \quad G_n^0 = G_0 \frac{\pi}{\sin \pi n} \left( \frac{\lambda^2}{\lambda_0^2} \right)^n \quad (n \rightarrow \infty)$$

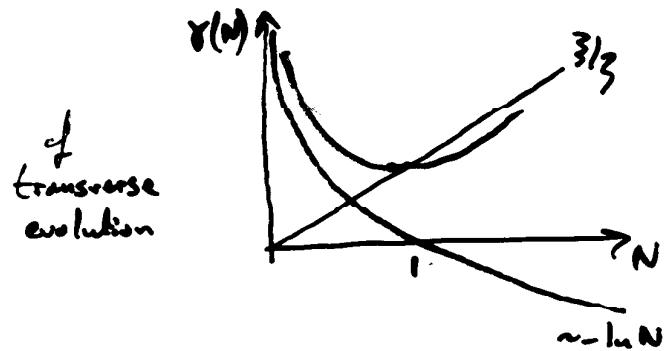
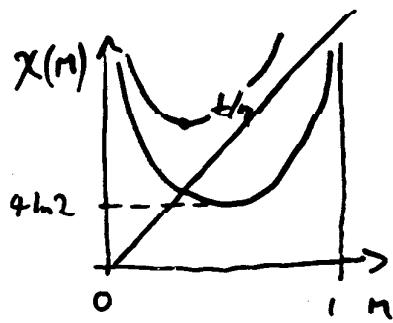
$\uparrow$   
non-perturbative  
scalar

$$(\sim Q^2, \text{ as } Q^2 \rightarrow 0, \sim \text{const as } Q^2 \rightarrow \infty) \quad (\text{Poles at } M = 0, 1)$$

At high energy,  $\frac{Q^2}{\lambda^2} \rightarrow \infty$ , saddle point is given by

$$t + \gamma^2 \chi'(M_s) \eta = 0$$

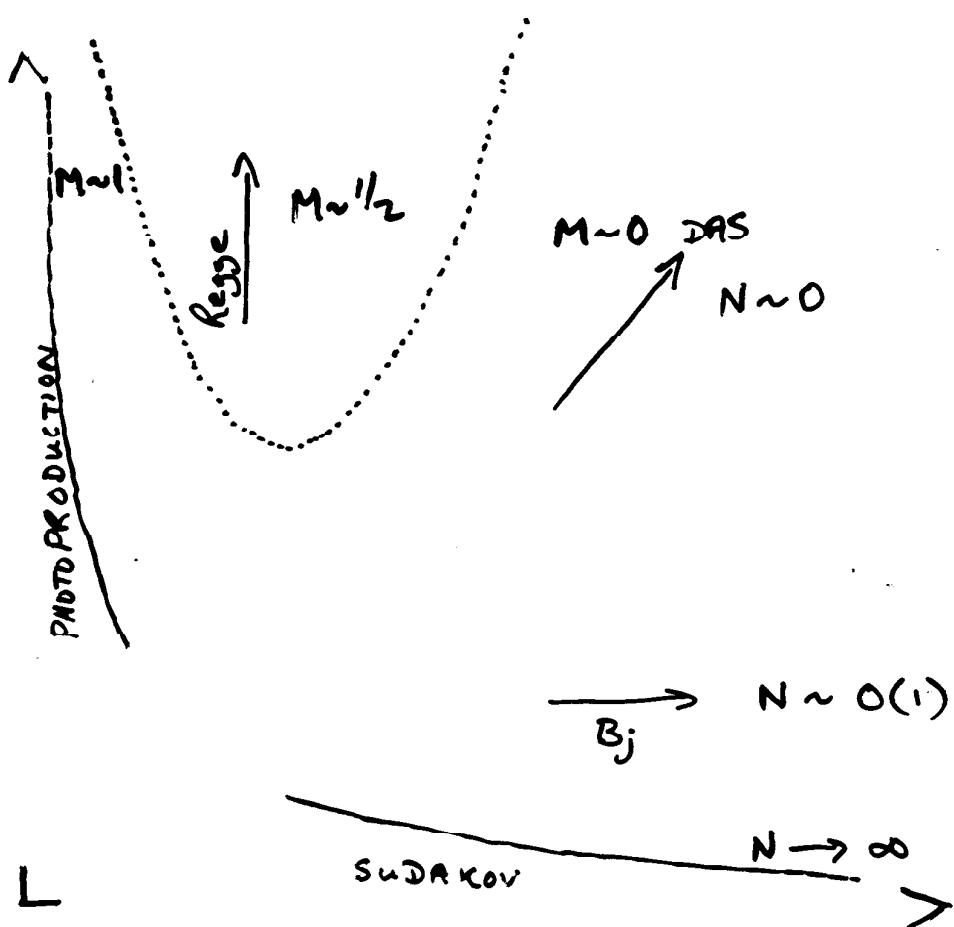
$\uparrow$   
 $\ln \frac{Q^2}{\lambda^2}$        $\uparrow$   
 $\ln \frac{\ln^2 x}{\ln^2 x_0}$



As  $\frac{\ln^2 x}{\ln^2 x_0}$  decreases from  $+\infty$  to  $-\infty$   
position of saddle moves from  $M=0$  to  $M=1$ ,  $\rightarrow G(x, Q^2)$  drops nontrivially

As  $\frac{\ln^2 x}{\ln^2 x_0}$  decreases from  $\infty$  to  $0$   
position of saddle moves from  $N=0$  to  $N=\infty$ ,  $\rightarrow g(x, Q^2)$  drops nontrivially

↗  
fig.



- $t$ -rge :  $\left\{ \begin{array}{l} N \text{ large} : \text{IR logs } (\log(1-x)) \text{ Kin. bdy.} \\ N \sim O(1) : B_j \text{ limit} \\ N \rightarrow 0 : \text{UV logs } (\log 1/x) \end{array} \right\}$  Double logs.
- $t$ -rge :  $\left\{ \begin{array}{l} M \rightarrow 0 : \text{UV logs } (\log Q^2_{\text{had}}) \\ M \sim 1/2 : \text{Regge limit} \\ M \rightarrow 1 : \text{IR logs } (\log N/\alpha^2) \text{ Kin. bdy.} \end{array} \right\}$

## High Energy Asymptotics

$$\begin{aligned} \gamma &= \ln \frac{\ln S^2/\lambda^2}{\ln S_0^2/\lambda^2} \\ &= \ln \frac{\ln'/\lambda}{\ln'/\lambda_0} \end{aligned}$$

- Regge Limit:  $\gamma \rightarrow \infty$ ,  $t$  fixed:

$$M_S \rightarrow 1_\infty, \chi(M_S) \rightarrow 4 \ln 2$$

$$F_2(x, Q^2) \sim N_0 \left( \frac{Q^2}{\lambda_0^2} \right)^{1/2} 2^{-1/2} (\ln'/\lambda)^{4 \cdot 8^{1/2}-1} + F_2^2(Q^2)$$

Universal logarithmic rise in  $x$ sec : "gluon plays the role of IP"

$\uparrow$   
Only one evolving parton,  
and  $\gamma_M$  has a minimum  
(at  $M = 1_\infty$ )

N.B. No powerlike rise  $x^{-\lambda}$   
ie no (perturbative) IP

Also (from coeff fns)

$$\lim_{x \rightarrow 0} \frac{F_L(x, Q^2)}{F_2(x, Q^2)} = \frac{2}{11}$$

Similar results for heavy quarks.

and sum rules

$$\int_0^\infty \frac{dQ^2}{Q^2} (Q^2)^m F_2(x, Q^2) = \left( \frac{3}{2} + \frac{1}{M(1-m)} \right) \int_0^\infty \frac{dQ^2}{Q^2} (Q^2)^{-m} F_L(x, Q^2)$$

- High Virtuality:  $\gamma \rightarrow \infty$  but  $t/\gamma \rightarrow 0$ :  $M_S \sim \sqrt[3]{\frac{2}{t}}$  (small)

and  $F_2(x, Q^2) \sim N_+ \frac{t^{4/3}}{\gamma^{5/3}} e^{2\sqrt{St}\gamma} - ?$

Smooth matching  
to DAS

- Low Virtuality:  $\gamma \rightarrow \infty$  but  $t/\gamma \rightarrow -\infty$ :  $M_S \sim 1 - \sqrt{\frac{2}{|t|}}$   
(photoprod.)

$$F_2(x, Q^2) \sim N_- \left( \frac{Q^2}{\lambda_0^2} \right) \frac{|t|^{2/3}}{\gamma^{5/3}} e^{2\sqrt{St}\gamma} - ?$$

Unitary!

So photoprod.  $x$ sec finite

(slower than any power)  
of  $\ln t$

▲ Infrared logs ( $\log \lambda_0^2/Q^2$ ) and quark mass effects

# Summation vs Resummation

Transverse Evolution  
(in  $Q^2$ )

Longitudinal Evolution  
(in  $S^2$ )

Summing  
Logs:

fixed coupling

$$\sum_0^\infty \frac{1}{n!} \alpha^n \delta_n^n (\ln Q^2)^n$$

Convergent Series

$$= (Q^2)^{\delta_n \alpha}$$

$$\sum_0^\infty \frac{1}{n!} \alpha^n \delta_n^n (\ln' x)^n$$

$$= (\ln' x)^{\delta_n \alpha}$$

$4 \ln 2 C_F \alpha_s / \pi$   
at  $M = \ln' x$

No  $B_j$  scaling:

disagrees with data....

BFKL IP:

disagrees with data:  
violates unitarity bounds.

Resumming  
Logs:

running coupling,  
ren. grp. eqns,  
asymptotic freedom.

$$\frac{\partial f_N}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \delta_N f_N$$

$$\rightarrow (\ln Q^2)^{2\delta_n/\beta_0}$$

$$\frac{\partial f_N}{\partial \ln S^2} = \frac{\alpha_s(S^2)}{2\pi} \delta_N f_N$$

$$\rightarrow (\ln' x)^{2\delta_n/\beta_0}$$

Can't be expanded as power  
series in logs : resummation

$B_j$  scaling violations:

agree with data

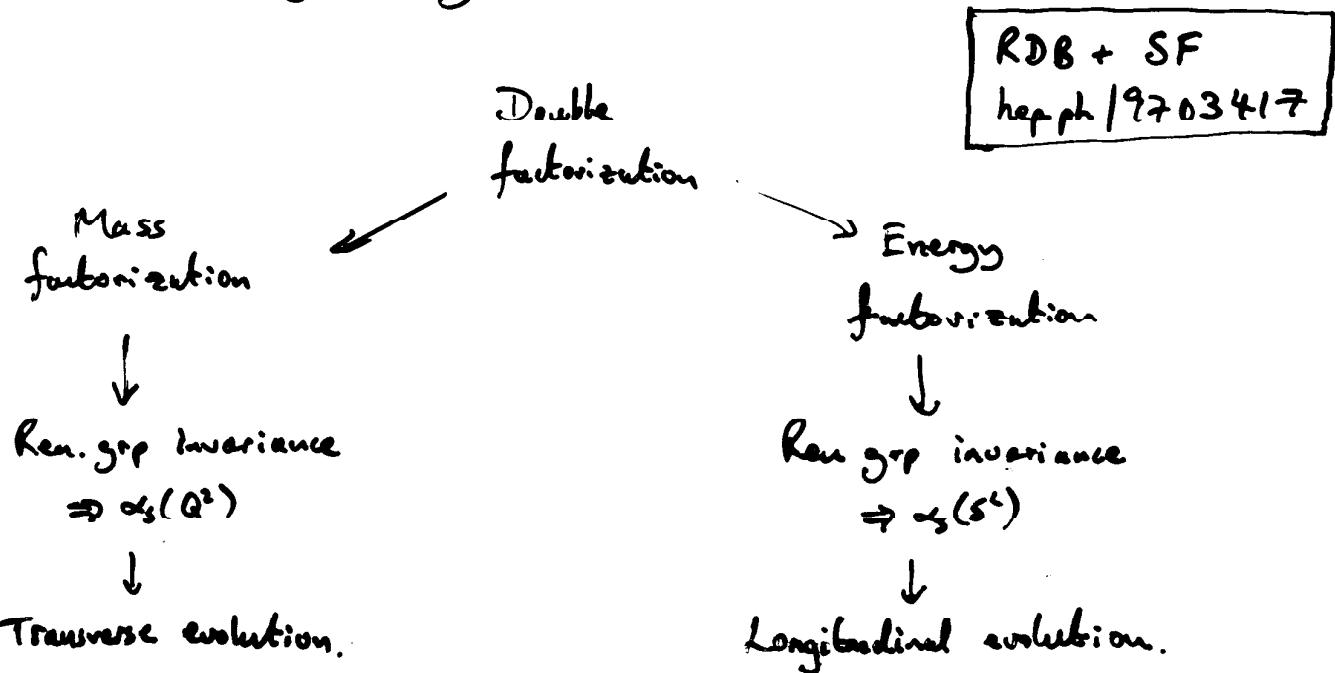
Gentle log rise (initiated by)  
soft IP

Unitarity O.K.

Asymptotic freedom the key property of pQCD  
both at high virtualities and at high energies

## Summary

- Have extended pQCD to inelastic ep scattering at high energies but low transverse momenta.



- Calculated LO longitudinal splitting fn (anom. dim.) } using  
and longitudinal coeff fns } Altarelli-Parisi  
method.

Now need NLO splitting fns and coeff functions. cf CFP  
▲ in energy fact. sch., ~~not~~ mass fact sch.

- High energy asymptotics:  $F_2 \sim \left(\frac{t}{\Lambda^2}\right)^{\gamma} \left(\ln^2 t/x\right)^{4\gamma^2 L^2 - 1}$   
But no need for a IP (no Regge trajectory with intercept  $> 1$ )  
Unitarity guaranteed

- The future:
  - diffractive processes, rap. gaps
  - photon-photon in  $e^+e^-$
  - hadronic processes (?)
  - etc. etc. etc. ....